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FARNBOROUGH, HANTS

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ON THE GEOMETRICAL CHANGES THAT OCCUR WHEN WINGS ARE ROTATED ABOUT AXES THAT ARE NEARLY NORMAL TO THE WINGS

by

C.H.E.WARREN

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ROYAL AIRCRAFT ESTABLISHMENT, FARNBOROUGH

On the Geometrical Changes that occur when Wings are rotated about Axes that are Nearly Normal to the Wings

by

C. H. E. Warren

R.A.E. Ref: Aero/7310

SUMMARY

This note merely presents the geometrical relationships that occur when the wings of an aircraft are rotated about axes that are nearly normal to the planes of the wings. Only elementary, but somewhat cumbersome, trigonometry is involved, but the relationships do depend upon precise definitions of the angles of sweepback and dihedral.

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1 Introduction

A number of recent projects have involved schemes in which the wings* have been rotated about hinges that are nearly normal to the planes of the wings. In such arrangements changes in sweepback, wing-body setting and dihedral occur. The variations of the wing-body setting and the dihedral with sweepback depend upon elementary, but somewhat cumbersome, trigonometry, but more particularly upon the precise manner in which the sweepback, wing-body setting and dihedral are defined. The purpose of this note is to present the geometrical relationships that occur by first defining precisely the relevant angles, and then deriving the various formulae relating them.

2 The definitions of sweepback and dihedral

The concepts of sweepback and dihedral are necessary purely to assist in describing the geometry of a pair of wings, and in the definitions of these terms recourse need not, and, in fact, should not, be made to other concepts such as the body, or the stream direction.

In the description of a pair of wings it is usual to specify some line in the wings, such as the line formed by the r^{th}-points of each chord, and called the r^{th}-chord line. Assuming that this r^{th}-chord line is straight for each wing, we now consider the laminar surface formed by the chords of one of the wings. With the root r^{th}-chord point as origin, let us take rectangular cartesian axes as follows:

x-axis forwards along the root chord,

z-axis downwards in the plane of symmetry,

y-axis to starboard.

The logical angles to take as the sweepback and the dihedral are those depicted in figure 1.

The sweepback of the r^{th}-chord line Λ_r is defined as the angle between the projection of the r^{th}-chord line on the xy-plane and the y-axis (See figure 1b).

The dihedral of the r^{th}-chord line δ_r is defined as the angle between the projection of the r^{th}-chord line on the yz-plane and the y-axis. (See figure 1a). If, however, the wing is untwisted, then the dihedral is independent of the choice of r, and in such cases the dihedral may be denoted simply by δ. (See figure 1c).

The above definitions are essentially the same as those given in reference 1. They are clearly convenient in that they can readily be obtained from a three-view drawing of the wings, as shown in figure 1. As phrased, the definitions apply only to 'straight-tapered' wings, but they could obviously be generalised to lead to definitions of the local sweepback and the local dihedral of wings of more general shape.

3 The apparent sweepback and apparent dihedral

As mentioned in section 2, the angles of sweepback and of dihedral can be obtained from a three-view drawing of the wings, provided, of course, that in the drawing the root chord is 'horizontal'. However, the three-view drawing of an aircraft is often made with the body datum

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* Of the two alternative terminologies - two separate wings together forming a pair of wings, or two half-wings together forming a complete wing - we shall adopt the former in this note.

line horizontal, the root chord being at a setting i to the body datum line. The angles obtained from such a drawing would not of course be the true angles of sweepback and dihedral, but they may be termed the apparent sweepback and apparent dihedral and denoted by the symbols Λ_r' δ_r' repectively. (See figure 2). The true sweepback and true dihedral may be obtained from them by elementary trigonometry. The relations are:

$$\tan \Lambda_r = \cos i \tan \Lambda_r' - \sin i \tan \delta_r' , \tag{3.1}$$

$$\tan \delta_r = \cos i \tan \delta_r' + \sin i \tan \Lambda_r' . \tag{3.2}$$

If we assume that the wing-body setting and the dihedral are small so that their squares and products may be neglected, the relations become:

$$\Lambda_r \simeq \Lambda_r' , \tag{3.3}$$

$$\delta_r \simeq \delta_r' + i \tan \Lambda_r' . \tag{3.4}$$

The inverse relations are, respectively:

$$\tan \Lambda_r' = \cos i \tan \Lambda_r + \sin i \tan \delta_r , \tag{3.5}$$

$$\tan \delta_r' = \cos i \tan \delta_r - \sin i \tan \Lambda_r , \tag{3.6}$$

$$\Lambda_r' \simeq \Lambda_r , \tag{3.7}$$

$$\delta_r' \simeq \delta_r - i \tan \Lambda_r \tag{3.8}$$

Sweepback and dihedral will be assumed to be true unless otherwise stated.

4 The exact geometry of a hinged wing

Let us now consider the geometry of a (starboard) wing which is hinged about an inclined axis through the root chord. (See figure 3). Let the hinge be situated at the r^{th}-point of the root chord, and, with this point as origin, let us take rectangular cartesian axes as follows:

X-axis forwards parallel to the body datum line,

Z-axis downwards in the plane of symmetry,

Y-axis to starboard.

The direction cosines of the root chord relative to these axes are $(\cos i, 0, -\sin i)$.

The direction ratios of the r^{th}-chord line, which we shall call the 'spar', are $(-\tan \Lambda', 1, -\tan \delta')$. Up to now we have used a suffix r to stress the dependence of sweepback and dihedral upon the particular r^{th}-chord line being considered. This suffix will now be dropped because we shall subsequently want to use suffices for other purposes.

The direction ratios of a line in the plane of the root chord and the spar, and perpendicular to the spar, and which we shall call a 'rib', can be obtained by elementary trigonometry, and are

$$[(\cos i \sec^2 \delta' + \sin i \tan \delta' \tan \Lambda'),\ (\cos i \tan \Lambda' - \sin i \tan \delta'),$$

$$(-\cos i \tan \delta' \tan \Lambda' - \sin i \sec^2 \Lambda')] .$$

Using the relations (3.1),(3.2),(3.5),(3.6) these direction ratios of the spar and rib may be expressed in terms of the true sweepback and true dihedral as follows:

spar: $[(-\cos i \tan\Lambda - \sin i \tan\delta),\ 1,\ (-\cos i \tan\delta + \sin i \tan\Lambda)]$

rib: $[(\cos i \sec^2\delta - \sin i \tan\delta \tan\Lambda),\ \tan\Lambda,\ (-\cos i \tan\delta \tan\Lambda - \sin i \sec^2\delta)]$

We are now in a position to investigate the change in geometry of the wing as it is rotated about the hinge. In such a rotation clearly the angles between the hinge axis and the spar, and between the hinge axis and the rib, will remain constant. Therefore, if the direction cosines of the hinge axis are (ℓ, m, n), we have

$$\ell(-\cos i \tan\Lambda - \sin i \tan\delta) + m + n(-\cos i \tan\delta + \sin i \tan\Lambda) = \text{constant} \tag{4.1}$$

$$\ell(\cos i \sec^2\delta - \sin i \tan\delta \tan\Lambda) + m \tan\Lambda + n(-\cos i \tan\delta \tan\Lambda - \sin i \sec^2\delta) = \text{constant} \tag{4.2}$$

5 The approximate geometry of a hinged wing

It is difficult to proceed further without making some approximations. We shall therefore assume that the wing-body setting and the dihedral are small, and that the inclined hinge is almost 'vertical'. In other words the squares and products of i, δ, ℓ, m will be neglected, and n will be treated as unity. With these approximations equations (4.1), (4.2) become

$$-\ell \sin\Lambda + m \cos\Lambda - \delta \cos\Lambda + i \sin\Lambda \simeq \text{constant} = c_1,\ \text{say} \tag{5.1}$$

$$\ell \cos\Lambda + m \sin\Lambda - \delta \sin\Lambda - i \cos\Lambda \simeq \text{constant} = c_2,\ \text{say} \tag{5.2}$$

Equations (5.1), (5.2) can be solved to give the variation of wing-body setting i and dihedral δ with sweepback Λ in terms of the angles of inclinations of the hinge axis ℓ, m, and the constants c_1, c_2. We obtain

$$i = \ell + c_1 \sin\Lambda - c_2 \cos\Lambda \tag{5.3}$$

$$\delta = m - c_1 \cos\Lambda - c_2 \sin\Lambda \tag{5.4}$$

5.1 Formulae pertaining to a particular sweepback

Suppose that the angles of inclination of the hinge ℓ, m are given, and that we know the wing-body setting i_0 and the dihedral δ_0 at a particular sweepback Λ_0. Then the constants c_1, c_2 can be determined in terms of i_0, δ_0, Λ_0 from equations (5.1), (5.2), and

substituting into equations (5.3), (5.4) we can determine the variation of wing-body setting i and dihedral δ with sweepback Λ in terms of the angles of inclination of the hinge ℓ, m, and the particular values i_0, δ_0, Λ_0. We obtain

$$i = \ell + (i_0 - \ell)\cos(\Lambda - \Lambda_0) - (\delta_0 - m)\sin(\Lambda - \Lambda_0) \tag{5.5}$$

$$\delta = m + (\delta_0 - m)\cos(\Lambda - \Lambda_0) + (i_0 - \ell)\sin(\Lambda - \Lambda_0) \tag{5.6}$$

Equations (5.5), (5.6) show that if $i_0 = \ell$ and $\delta_0 = m$, then i, δ do not vary as Λ varies. Now $i_0 = \ell$ implies that the hinge axis is perpendicular to the root chord (see figure 3b), and $\delta_0 = m$ implies that the hinge axis is perpendicular to the r^{th}-chord line or spar (see figure 3a). If both these relations hold then clearly the hinge line is perpendicular to the 'plane of the wing' (that is, the plane formed by the chords of the wing, assuming it to be untwisted), and if the wing rotates about an axis perpendicular to itself, then clearly the wing-body setting and the dihedral will not vary.

5.2 Formulae pertaining to two representative sweepbacks

Suppose that the wing-body settings i_L, i_H, and the dihedrals δ_L, δ_H at two representative sweepbacks Λ_L, Λ_H (the low and the high) are given. By substituting into equations (5.1), (5.2) we obtain four equations which can be solved for ℓ, m, c_1, c_2. We obtain

$$\ell = \tfrac{1}{2}(i_H + i_L) - \tfrac{1}{2}(\delta_H - \delta_L)\cot\left[\tfrac{1}{2}(\Lambda_H - \Lambda_L)\right] \tag{5.7}$$

$$m = \tfrac{1}{2}(\delta_H + \delta_L) + \tfrac{1}{2}(i_H - i_L)\cot\left[\tfrac{1}{2}(\Lambda_H - \Lambda_L)\right] \tag{5.8}$$

$$c_1 = \tfrac{1}{2}(i_H - i_L)\frac{\cos\left[\tfrac{1}{2}(\Lambda_H + \Lambda_L)\right]}{\sin\left[\tfrac{1}{2}(\Lambda_H - \Lambda_L)\right]} + \tfrac{1}{2}(\delta_H - \delta_L)\frac{\sin\left[\tfrac{1}{2}(\Lambda_H + \Lambda_L)\right]}{\sin\left[\tfrac{1}{2}(\Lambda_H - \Lambda_L)\right]} \tag{5.9}$$

$$c_2 = \tfrac{1}{2}(i_H - i_L)\frac{\sin\left[\tfrac{1}{2}(\Lambda_H + \Lambda_L)\right]}{\sin\left[\tfrac{1}{2}(\Lambda_H - \Lambda_L)\right]} - \tfrac{1}{2}(\delta_H - \delta_L)\frac{\cos\left[\tfrac{1}{2}(\Lambda_H + \Lambda_L)\right]}{\sin\left[\tfrac{1}{2}(\Lambda_H - \Lambda_L)\right]} \tag{5.10}$$

Substituting for c_1, c_2 from equations (5.9), (5.10) into equations (5.3), (5.4) we can determine the variation of wing-body setting i and dihedral δ with sweepback Λ in terms of the particular values i_L, i_H; δ_L, δ_H; Λ_L, Λ_H. We obtain

$$i = \ell + \tfrac{1}{2}(i_H - i_L)\frac{\sin\left[\Lambda - \tfrac{1}{2}(\Lambda_H + \Lambda_L)\right]}{\sin\left[\tfrac{1}{2}(\Lambda_H - \Lambda_L)\right]} + \tfrac{1}{2}(\delta_H - \delta_L)\frac{\cos\left[\Lambda - \tfrac{1}{2}(\Lambda_H + \Lambda_L)\right]}{\sin\left[\tfrac{1}{2}(\Lambda_H - \Lambda_L)\right]} \tag{5.11}$$

$$\delta = m + \tfrac{1}{2}(\delta_H - \delta_L)\frac{\sin\left[\Lambda - \tfrac{1}{2}(\Lambda_H + \Lambda_L)\right]}{\sin\left[\tfrac{1}{2}(\Lambda_H - \Lambda_L)\right]} - \tfrac{1}{2}(i_H - i_L)\frac{\cos\left[\Lambda - \tfrac{1}{2}(\Lambda_H + \Lambda_L)\right]}{\sin\left[\tfrac{1}{2}(\Lambda_H - \Lambda_L)\right]} \tag{5.12}$$

where ℓ, m, are given by equations (5.7), (5.8).

5.3 Formulae pertaining to a 'vertical' hinge

Suppose that we know we know the constants c_1, c_2 for an inclined hinge, and also the constants c_1', c_2' for a 'vertical' hinge, the wing being at the same sweepback Λ, wing-body setting i, dihedral δ in the two cases. Then for the inclined hinge equations (5.1), (5.2) hold, but for the vertical hinge we have

$$-\delta \cos\Lambda + i \sin\Lambda = c_1' \qquad (5.13)$$

$$-\delta \sin\Lambda - i \cos\Lambda = c_2' \qquad (5.14)$$

Eliminating i, δ between equations (5.1), (5.2), (5.13), (5.14) we obtain

$$c_1 = c_1' - \ell \sin\Lambda + m \cos\Lambda \qquad (5.15)$$

$$c_2 = c_2' + \ell \cos\Lambda + m \sin\Lambda \qquad (5.16)$$

which express the constants for the inclined hinge in terms of those for the vertical hinge.

REFERENCE

No.	Author	Title, etc.
1	British Standards Institution	British Standard Glossary of Aeronautical Terms B.S.185: Part 1: 1950

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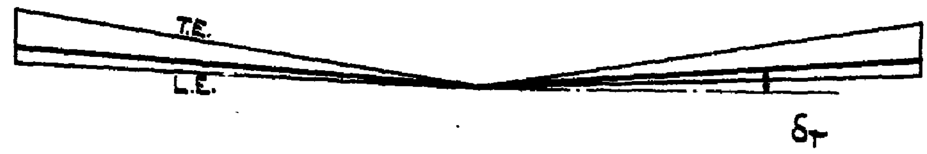
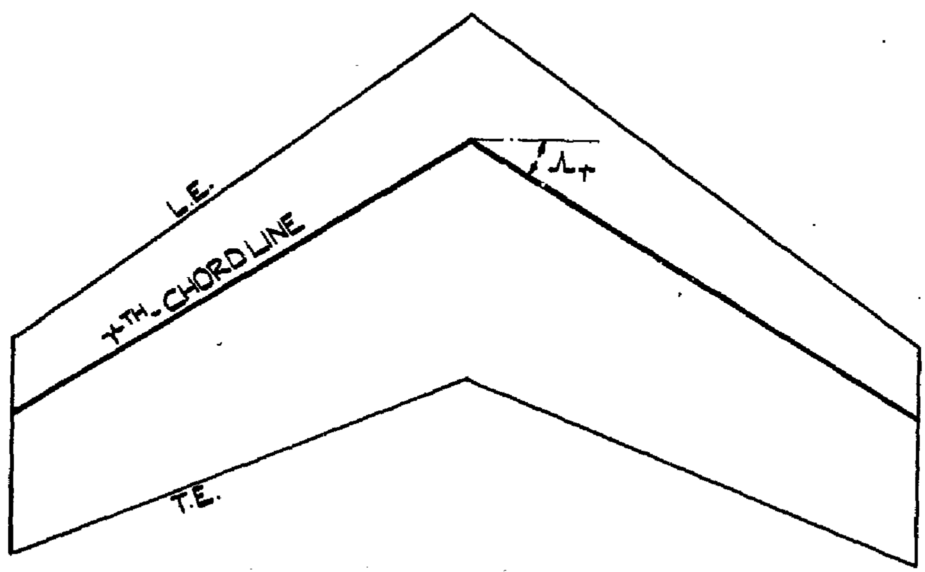


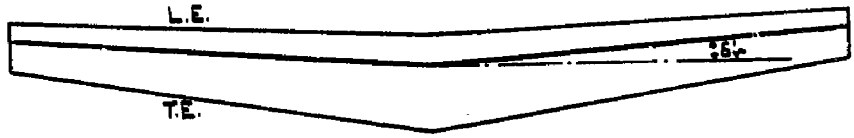
(a) REAR VIEW OF WINGS IF TWISTED.



(b) PLAN VIEW OF WINGS.

(c) REAR VIEW OF WINGS IF UNTWISTED.

FIG 1. (a-c) DEFINITIONS OF SWEEPBACK AND DIHEDRAL.



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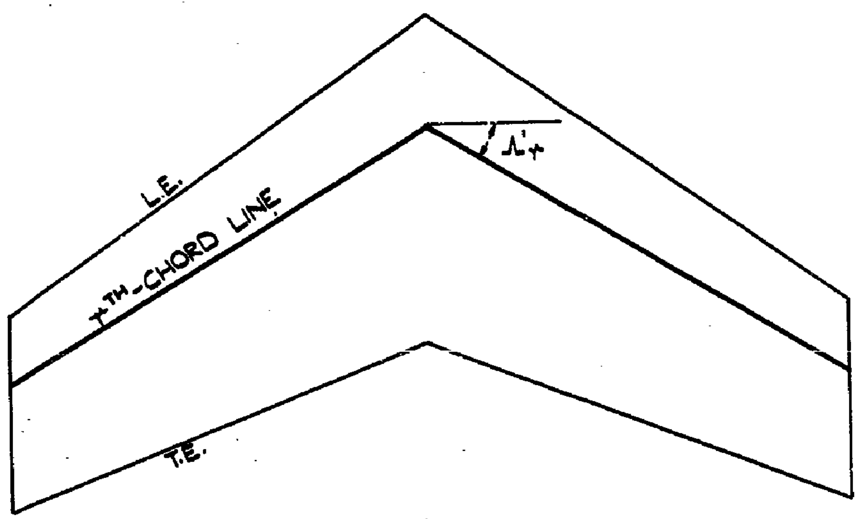
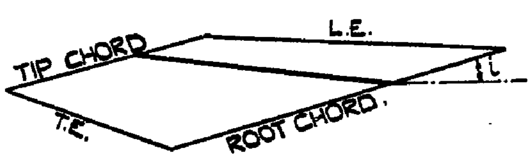
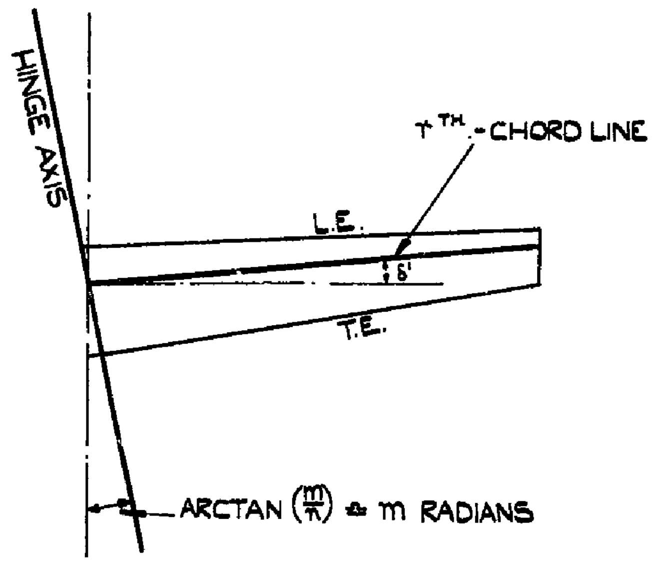


(b) PLAN VIEW OF WINGS.

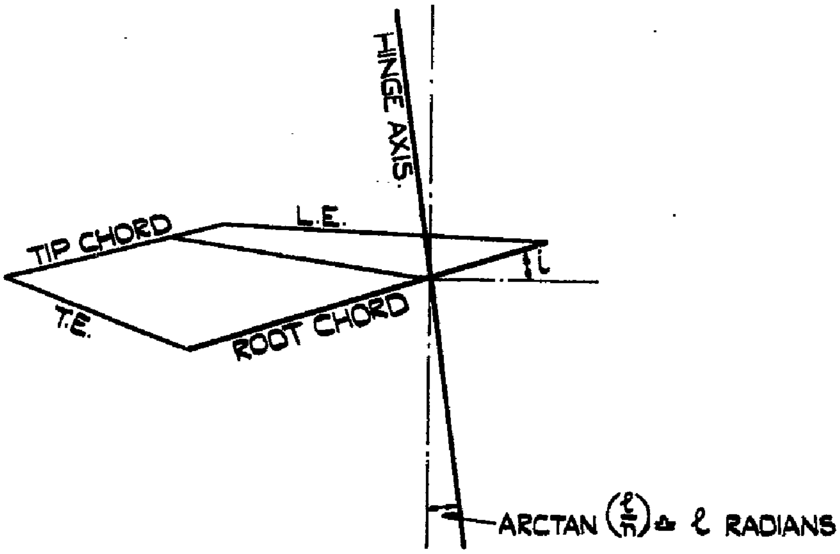


(c) SIDE VIEW OF WINGS.

FIG. 2.(a-c) GEOMETRY OF WINGS AT A SETTING TO A BODY.



(a) REAR VIEW OF STARBOARD WING



(b) SIDE VIEW OF STARBOARD WING.

FIG. 3.(a & b) GEOMETRY OF A HINGED WING

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